

# Looking Into Noise-Figure Measurement Uncertainty

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With today's applications requiring lower and lower noise-figure amplifiers, noise-source uncertainty becomes a bigger and bigger concern.

**M**icrowave modeling software has enabled extremely low noise-figure (NF) amplifiers to become commonplace. There was a time when low-noise amplifiers (LNAs) below 0.7 dB were found only on specialized, expensive systems such as MRI machines or highly sensitive scientific radiometers. However, within many companies, the basic understanding of NF measurement has not kept up with the capability to manufacture LNAs. Specifications of 0.7 dB are being written for NFs, yet often the devices are being tested with a NF setup in which the noise source itself has an uncertainty of

0.3 dB without even considering other noise that may be added by the instrumentation. Your microwave amplifier company is rolling along smoothly, but when the NF test stand returns from its annual calibration, all the products tested on it suddenly do not meet their NF specifications. Much of the resulting panic could have all been avoided with a little uncertainty planning.

How the noise source contributes to NF uncertainty and ways in which this can be minimized are best appreciated by reviewing a few mathematical expressions involving NF. The fundamental definition of noise factor (F) is the ratio of the signal-to-noise power ratio (SNR) at the input of the DUT to that at the output:

$$F = \text{SNR}_{\text{in}} / \text{SNR}_{\text{out}} \quad (1)$$

NF is related to F by:

$$\text{NF} = 10 * \log_{10}(F) \text{ dB} \quad (2)$$

Noise power can be expressed as a function of absolute temperature:

$$P_N = kTB \quad (3)$$

where:  $P_N$  = noise power in watts  
 $k$  = Boltzmann's constant,  
 $1.38 \times 10^{-23}$

$B$  = bandwidth in Hertz

Because of this relationship, it is common to define F and NF in terms of an effective, or equivalent, noise temperature:

$$F = 1 + T_H / T_0 \quad (4)$$

where:  $T_H$  = equivalent noise temperature  
 $T_0$  = reference temperature 290°K

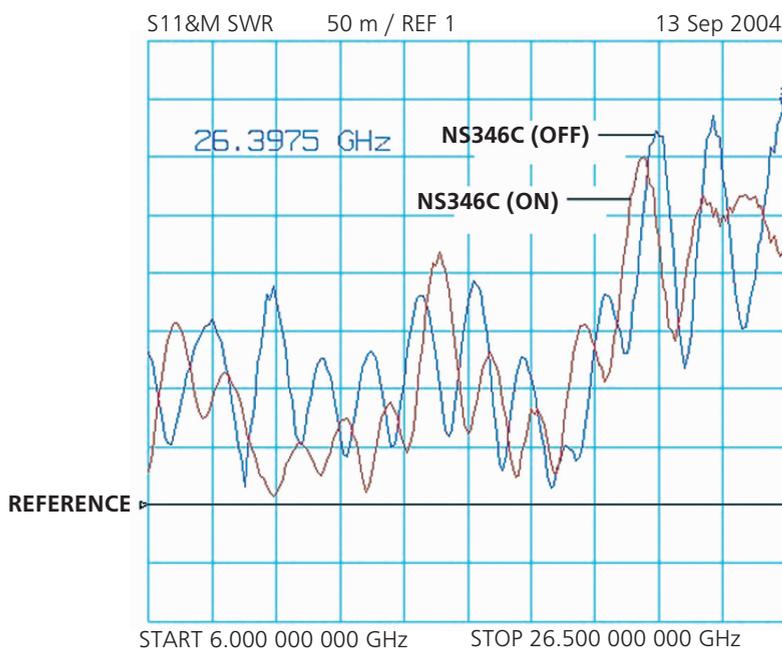


Figure 1. S11 Data for Nominal 15-dB Noise Source

The excess noise ratio (ENR) is a related value and often expressed in decibels:

$$ENR_{dB} = 10 \cdot \log_{10}(T_H/290 - 1) \quad (5)$$

NF can be expressed in terms of ENR and a measured DUT power ratio:

$$NF_{(dB)} = ENR_{(dB)} - 10 \log_{10}(Y_{Fact} - 1) \quad (6)$$

where:  $Y_{Fact} = N_2/N_1$

$N_2$  = the measured power output with the noise source on

$N_1$  = the measured power output with the noise source off

Most NF test stands use a noise source calibrated to absolute ENR amplitudes at discrete frequency points. These values are plugged into electronic storage in the NF meter.

The NF meter automatically makes power measurements with the noise source on and with the noise source off and computes the NF reading of the DUT using the stored noise-source ENR value. The NF is a simple sum function of ENR, the first term of Equation 6. This means that any uncertainty of ENR is directly passed on to the uncertainty of the NF measurement.

There also is an uncertainty associated with the Y-factor term. This is a result of the uncertainty of the NF measurement instrumentation. The total NF uncertainty, assuming perfect match, is the sum of the instrumentation and noise-source uncertainties:

$$U_{NF} = U_{NS} + U_{Inst} \quad (7)$$

Of course, a perfect match doesn't exist. The additional uncertainty caused by the mismatch depends on the complex impedances of the noise source in both its on and off states, of the two DUT ports, and of the NF meter input port. The relationships are interdependent, so it is too simplistic to express the effect by adding another term to Equation 7.

The technique of characterizing the mismatches and canceling them out is called vector error correction (VEC). The associated equations get very large and unwieldy. VEC, although a valuable technique, will not be discussed due to the mathematical complexity and because:

- Noise-source matches are extremely good, typically better than 1.25:1 VSWR.
- The match of the DUT typically is very good if it is a low-NF device.
- The match of today's measurement instruments is very good.

The fact that mismatches are quite low in practice translates to an extremely low error caused by the mismatch in relation to the uncertainty of the noise-source ENR alone. When using VEC, it also is advisable to automate the vector impedance measurement process and write the mathematical conversions into software. Otherwise, the sheer size of the equations

Frequency (GHz)	Noise Temperature (°K)	ENR (dB)
0.01	7700 ±130	14.074 ±0.078
0.10	7850 ±140	14.160 ±0.079
1.00	7170 ±100	13.752 ±0.065
3.00	6980 ±130	13.631 ±0.081
6.00	7620 ±120	14.028 ±0.070
9.00	9210 ±140	14.880 ±0.068
12.00	10370 ±230	15.410 ±0.098
15.00	10890 ±200	15.631 ±0.080
18.00	9770 ±260	15.150 ±0.120
21.00	11030 ±420	15.680 ±0.170
24.00	12430 ±580	16.220 ±0.210
26.50	11190 ±690	15.750 ±0.270

**Table 1. 15-dB Noise-Source Calibration Data**  
Courtesy of NPL

can lead to a high likelihood of human error, which ironically will produce more error than if the mismatch simply were ignored. As the correction is so subtle, calculation errors may not be obvious and go undetected even under close scrutiny.

### Standards and Calibration

The noise source and the NF instrumentation are separate entities as are the processes that calibrate them. A quality instrumentation calibration can result in an uncertainty of 0.12 dB through the X band (12.4 GHz) and perhaps 0.15 dB through the K band (26.5 GHz). There tends to be much more variance in the noise-source uncertainty.

Noise sources are in the family of test and measurement devices known as calibration standards. This means they are calibrated to an absolute level traceable to an ultimate standard. In the United States, calibration standards are kept at the National Institute of Standards and Technology (NIST). In England, the organization is the National Physical Laboratory (NPL). Organizations such as NIST and NPL are referred to as national labs.

You might be familiar with the gauge pin box that is standard fare for any QA department. These mechanical standards are tightly manufactured and measured against another standard. That standard might have been calibrated directly by a national lab. The calibration report from the national lab will state an uncertainty.

This device now is what might be known in the industry as a gold standard. If that standard is used as a calibration standard, the device it is calibrating will be known as a silver standard. The next in line is a bronze standard. The silver standard is not as accurate as the gold standard, and the bronze is not as accurate as the silver. Gauge pins and noise sources are similar in this fashion.

An added uncertainty is a result of the transfer instrumentation:

$$U_{DUT} = U_{Standard} + U_{Transfer} \quad (8)$$

Frequency (GHz)	Noise Temperature (°K)	ENR (dB)
0.01	1184 ±16	4.890 ±0.077
0.10	1202 ±16	4.978 ±0.077
1.00	1235 ±14	5.128 ±0.064
3.00	1265 ±18	5.267 ±0.081
6.00	1263 ±16	5.255 ±0.070
9.00	1287 ±16	5.361 ±0.069
12.00	1258 ±22	5.237 ±0.097
15.00	1314 ±19	5.479 ±0.080
18.00	1348 ±30	5.620 ±0.120

**Table 2. 5-dB Noise-Source Calibration Data**  
Courtesy of NPL

Each generation removed from the gold standard gets a little worse. Most QA documentation requires all calibration standards be traceable to national labs.

Too often, it is assumed that traceability means accuracy. A device 100 generations away from a gold standard still has traceability, yet it hardly will be accurate.

Noise sources are calibrated at the national laboratory using actual hot and cold loads. These two loads will have different power-measurement readings used as two points to define a straight-line curve. The power of the noise source being calibrated is referenced to this line, and its noise temperature is determined ( $T_H$  of Equation 5). However, the uncertainty of the  $ENR_{dB}$  is given by:

$$U_{ENR(dB)} = [10/\ln(10)] * [U_{TH} / (T_H - 290)] \quad (9)$$

Examples of calibration reports are given in **Table 1**

and **Table 2**. When comparing the two noise-source uncertainties at, for example 9 GHz, the 15-dB noise-source ENR has a much higher temperature uncertainty (140°K vs. 16°K), yet on the decibel scale, they are relatively close (0.068 dB vs. 0.069 dB). For this reason, scientists traditionally have used the noise temperature for both the DUT and the noise source as opposed to using the decibel NF and  $ENR_{dB}$ , respectively, because it offers a clearer picture of system uncertainty.

The disparity of temperature uncertainty results from the large extrapolation required because it obviously is impossible to have a load at a temperature of 9,200°K. The uncertainties represent typical uncertainties, not guaranteed uncertainties.

The nominal noise-source ENR plays a role in the overall NF measurement accuracy. The optimum ENR of the noise source depends on the expected NF of the DUT.

If the expected NF is high, the measured difference of the off and on noise-source states will be too small to discern accurately given the DUT's comparatively large amount of self-generated thermal noise. However, if the expected NF is very low and if too high a noise source is used, then the two measured values may have such disparate amplitudes that nonlinear dynamic-range issues may compromise accuracy.

Depending on how crucial the measurement-uncertainty window needs to be, the designer can mathematically calculate the theoretical best ENR although this process can be quite exhaustive. **Table 3** indicates a quick rule of thumb for ENR vs. expected NF. As shown, a low-ENR noise source should be used for low expected NFs. The low-ENR noise source also has lower ENR uncertainty, further improving accuracy.

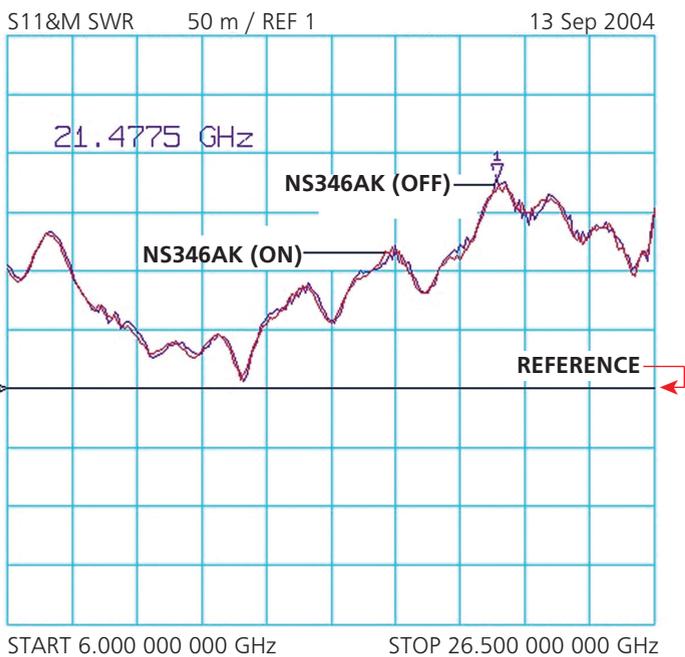
There is one additional benefit: the improved match of the lower ENR noise source over the higher. The match is better because the low-power noise source is constructed with a larger attenuator pad at the output. The return loss theoretically is 2x this value, so a higher value pad makes a big impact on return loss or match.

It isn't just the magnitude of improvement. Rather, the delta of match between off and on states is most critical.

The noise diode circuit is very reflective in the off state, resulting in a huge on vs. off delta. A high pad value absorbs most of this difference where a smaller value can't.

**Figure 1** and **Figure 2** depict S11 data from 6 GHz to 26.5 GHz, showing the difference between the Micronetics NS-346C noise source and the NS-346AK noise source in both the off and on states. Being cognizant of the on/off match still is important even if you do not perform vector analysis.

Once the noise source has been selected, you need to examine the calibration. The ultimate is to get a calibration directly from NIST. The cost and lead time, although quite reasonable for the quality of calibration, are substantially higher than those from a local calibration house or noise-source manufacturer that uses a reference



**Figure 2. S11 Data for Nominal 6.5-dB Noise Source**

Expected NF	Nominal ENR
0 to 10 dB	6 dB
10 to 20 dB	10 dB
20 to 35 dB	15 dB

**Table 3. ENR vs. Expected NF**

ing from a reference standard is relatively quick and easy.

It is important to understand what type of standard is being used when noise sources are sent out for calibration. The best-case scenario is that your noise source was calibrated with a standard directly calibrated by a national lab. A calibration house should be able to provide this information.

Understanding the elements that contribute to NF measurement uncertainty is very important. As the noise-source uncertainty may be the biggest contributor, pay special attention to it. Absolute specifications need to be written with some basis in reality while keeping a close eye on the uncertainty of the test stand that verifies the NF specifications. With a little effort and contentiousness, NF measurement accuracy can be maximized with a high

noise standard to calibrate noise sources. The difference in cost exists because the process of calibrating

degree of repeatability and no surprises that hold up product shipments.

### References

1. *National Physical Laboratory Calibration Report*, November 2003.
2. Robbins, P., "Using Noise for RF Receiver Built-in Test Applications," *Microwave Journal*, February 2004.
3. Craig, T.M., Noise Source Module For Microwave Test Systems, U.S. Patent 6,268,735, 2001.

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